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DEVELOPING A CONCRETE INEQUALITY CONDITION FOR TAYLOR'S HYPOTHESIS IN COMMON TURBULENT ATMOSPHERIC FLOWS

BY

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Abstract. In this paper, the problem of applying stochastic methods to the analysis of standard turbulent atmospheric flow is considered. The nature of randomness and stochasticity is considered and contrasted to the deterministic, yet chaotic, nature of atmospheric turbulent flow. Despite the many difficulties inherent in using a statistical interpretation for turbulence, a precise inequality condition can be established, based on Taylor's frozen flow hypothesis, to show whether or not a particular instance of turbulent flow can be considered in a stochastical manner in terms of individual meteorological measurements.

Keywords: turbulence; randomness; stochastic; advection.

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1. Introduction

In many scientific fields, the word "random" has become almost synonymous with the word "stochastic": however, "stochastic" refers to a random process rather than to "randomness-in-itself". Stochasticity is promoted as an operating mode or method in many scientific fields and disciplines, while the enormous number of degrees of freedom and entities in atmospheric flows give "the impression" of stochasticity and randomness; however, almost the entire scientific body of work that has been written regarding the atmosphere could be used to show, first of all, that it exhibits memory-like properties. From most perspectives, the atmosphere cannot be said to be random or memoryless: atmospheric events take place because of present and past factors, however chaotically they appear to fluctuate. The presence of a quality which might be, even roughly, likened to memory in the analysis of atmospheric phenomena then denies all possibilities for such processes to be stochastic or markovian in nature (Markov, 1954; Dodge, 2006). Rigorous mathematical arguments for the non-randomness of atmospheric flow can be convoluted and complicated, and there does not seem to be a solid scientific consensus yet regarding the specifics of these arguments – however, the mere fact that atmospheric turbulence seems to structure itself into multiple splitting coherent bodies named "vortices" shows that the entire process is not random. And, of course, it should be highlighted that many chaotic processes cannot be considered random, and chaos does not necessarily imply randomness.

In the scientific field of pure randomness, the most common example of "truly random units" is found as a product of what is named "algorithmic randomness". Algorithmic randomness, in general, can refer to the study of randomly produced individual elements in sample spaces, these being the set of all possible infinite binary sequences (Downey, 2010). An algorithmically random element passes basically all devised tests for randomness (Downey, 2010). As different types of algorithms are sometimes considered, ranging from algorithms with specific bounds on their running time to algorithms which may "ask questions" of a machine, there are different notions of randomness. The most common of these is known as Martin-Löf randomness, but stronger and weaker forms of randomness also exist (Downey, 2010).

It is important to distinguish algorithmic randomness from stochastic randomness; unlike algorithmic randomness, which is defined for computable (and thus deterministic) processes, stochastic randomness is usually said to be a property of a sequence that is a priori known to be generated by an independent identically-distributed equiprobable stochastic process. Thus, we must not to confuse "stochastic randomness" with "randomness-as-such"; in any case, just because it is possible to conceive of, and generate, algorithms that produce "as-random-as-possible" sequences of numbers, this does not automatically imply that such algorithms might be represented by real, physical processes.

Furthermore, despite what we know of the atmosphere, there is even further cause for doubt regarding such possibilities in atmospheric processes.

2. The Taylor Hypothesis

Despite all of these potential disadvantages, it is entirely possible to conceive of certain limited cases where stochastic methods can be employed in the study of turbulent flow. Many experimental investigations of turbulent velocity fields often invoke Taylor's hypothesis, also known as the "frozen flow hypothesis", to determine the spatial structure of turbulent flow based on time-resolved single-point measurements. For the approximations to be valid, it is crucial that the turbulent fluctuations themselves are supposed to be quite small compared to the mean velocity; in other words that the turbulence intensity has to be relatively low (Wilczek, 2014). Thus, Taylor's frozen flow hypothesis is the assumption that, under certain circumstances, advection resulted from turbulent circulations themselves is very small and that therefore the advection of a field of turbulence past a fixed point can be taken to be entirely due to the mean flow.

One of standard definitions for the given circumstances:

$$\frac{u}{U} \ll 1 \quad (1)$$

where u is defined as the velocity of a turbulent eddy in the flow and U is mean flow velocity (Glossary of Meteorology, 2019). This definition, however, raises a number of questions; one of these questions is: which eddy velocity does the definition entail? Another common definition for frozen flow validity is:

$$U^{'} < \frac{1}{2}\langle U \rangle \quad (2)$$

In this manner, the hypothesis is valid when the flow speed variation due to turbulence is less than half of the mean of the flow speed. However, the term $\frac{1}{2}$ appears to be chosen arbitrarily. In the attempt to construct a more rigorous definition, we must find a rigorously-determined inequality condition. It is found that Taylor's frozen flow hypothesis, without closure and under general assumptions, is "valid up to time scales smaller than the correlation time scale of temporal velocity correlation function" (Bahraminasab, 2008).

Many different versions of a temporal velocity correlation function can be constructed a priori, however a simple example can be found:

$$B(\tau) = a^2 e^{-\left|\frac{\tau}{\tau_0}\right|} \quad (3)$$

where a is a given constant, and τ_0 is the correlation time (Tatarskii, 1961). The physical meaning of this interval of time is found in:

$$B(0) = a^2 \tag{4a}$$

$$B(\tau_0) = \frac{a^2}{e} \tag{4b}$$

The general definitions of the temporal correlation function and temporal structure function for a parameter in a turbulent flow:

$$B(\tau) = \langle f(t) f^*(\tau + t) \rangle \tag{5a}$$

$$D(\tau) = \langle [f(\tau + t) - f(t)]^2 \rangle \tag{5b}$$

but in the case of real values, we will obtain:

$$B(\tau) = \langle f(t) f(\tau + t) \rangle \tag{6}$$

In order to use these temporal correlation and structure functions, and to employ the statistics presented so far, we must consider that the chosen atmospheric parameter is random. This is problematic in the context of the previous discussion; however, in choosing wind speed as a parameter at a fixed altitude, and in the context of relatively-calm atmospheric conditions and during a short temporal interval which we shall name τ_T, this approximation can hold quite well.

The following relation is also found (Tatarskii, 1961):

$$B(0) = \frac{D(\tau)}{2} + B(\tau) \tag{7}$$

Then:

$$B(0) = \frac{D(\tau_0)}{2} + B(\tau_0) \tag{8}$$

or:

$$\frac{D(\tau_0)}{2} + B(\tau_0) = ct. \tag{9}$$

The equations above are thus true irrespective of τ_0:

$$B(\tau_0) = \frac{B(0)}{e} = \frac{1}{e}\left[\frac{D(\tau_0)}{2} + B(\tau_0)\right] \tag{10}$$

The result is:

$$2(e-1)B(\tau_0) = D(\tau_0) \tag{11}$$

thus:

$$2(e-1)\langle U(t)U(\tau_0+t)\rangle = \langle [U(\tau_0+t)-U(t)]^2\rangle \tag{12}$$

In this case, Taylor's hypothesis is true in a timeframe $\tau_T \le \tau_0$. Of course, this is viable for the simple correlation function that we have chosen beforehand.

We continue with:

$$2(e-1)\sum_{i=1}^{n} U(t_i)U(t_i+\tau_T) \ge \sum_{i=1}^{n}[U(t_i+\tau_T)-U(t_i)]^2 \tag{13}$$

thus:

$$\frac{\sum_{i=1}^{n} U(t_i)U(t_i+\tau_T)}{\sum_{i=1}^{n}[U(t_i+\tau_T)-U(t_i)]^2} \ge \frac{1}{2(e-1)} \tag{14}$$

The inequality is logically given by:

$$B_U(\tau_T) \ge \frac{a^2}{e} \tag{15}$$

Because, the smaller the τ, the larger the correlation function, and closer to $B(0)$.

It follows that Taylor's hypothesis is valid in the timeframe when the velocity field follows the upper inequality, or when:

$$2(e-1)B_U(\tau_T) \ge D_U(\tau_T) \tag{16}$$

It is unclear, however, how the structure function behaves strictly in terms of the inequality; a more simplified and accurate condition might be derived from Eq. (4b):

$$\sum_{i=1}^{n} U(t_i)U(t_i+\tau_T) \ge \frac{1}{e}\sum_{i=1}^{n} U(t_i)^2 \tag{17}$$

Given the fact that the division of the two terms is proportional to e, it is a given that a velocity field exhibiting very low variations during the τ_T interval is "Taylor-compliant". In order for such a velocity field to fail this inequality, it would have to present high enough velocity differences such that:

$$\sum_{i=1}^{n}[eU(t_i)U(t_i+\tau_T)-U(t_i)^2] \ge 0 \tag{18}$$

In principle, this also implies that a velocity field whose intensity decreases significantly over τ_T does not pass the inequality, however this might point to the effects of turbulent dissipation. At the same time, a velocity field whose intensity increases over τ_T passes the inequality, which is just what one might expect given a potential dominance of mean flow effects.

3. Conclusion

The notion of the structure function of a parameter in a turbulent flow can be expanded so as to connect various equations regarding the structure and inhomogeneity of that parameter field in a turbulent flow; however, central to the following theory is the idea that many of these parameters are conservative passive additive quantities. It is implied in the statistical methods used in this work that very fast wind speed measurement is necessary in order to construct a meaningfully-large sample of measurements for the summation in Eq. (18) and others. In any case, we conclude by stating that statistical methods can be used successfully in the study of atmospheric turbulence, regardless of the fact that, for the most part, the atmosphere is most accurately considered a deterministic (yet incredibly complex) medium.

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DEZVOLTAREA UNEI CONDIȚII STRICTE PENTRU IPOTEZA LUI TAYLOR ÎN CURGERI ATMOSFERICE TURBULENTE

(Rezumat)

În acest articol, sunt prezentate tehnici stohastice de analiză a curgerilor turbulente atmosferice standard. Natura proceselor aleatorii pure și a stohasticității este cercetată şi comparată cu natura deterministă, dar haotică, a curgerilor turbulente atmosferice. În ciuda dificultăților de-a implementa o interpretare statistică a curgerilor turbulente, se stabileşte o restricţie, sub forma unei inegalități bazate pe ipoteza lui Taylor, pentru a arăta validitatea unei astfel de interpretări în cazul măsurătorilor meteorologice.